Symmetry realization of the textures of neutrino mass matrix with one vanishing minor and vanishing trace using Froggatt-Nielsen Mechanism

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December 6, 2021

Introduction

- From the standard model of particle physics, we find that the neutrinos ν_e, ν_μ and ν_τ are massless.
- Masses and mixing of three flavors of neutrino can be described by a (3×3) complex symmetric Majorana mass matrix M_ν which is parametrized [1] by a total of nine parameters $(m_1, m_2, m_3, \theta_{12}, \theta_{13}, \theta_{23}, \delta, \alpha, \beta)$.
- Out of these nine parameters only five of them are measured by neutrino oscillation experiments. They are the three mixing angle$(\theta_{12}, \theta_{13}, \theta_{23})$ and two mass squared differences $(\Delta m^2_{21}, \Delta m^2_{32})$

Formalism

- The neutrino mass matrix M_ν is given by

$$M_\nu = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T \tag{1}$$

  Where V is the PMNS matrix which is parametrized as $V = UP_\nu$, where $P_\nu = diag(1, e^{i\alpha}, e^{i(\beta+\delta)})$ where α and β are two majorana CP-phases and δ is the Dirac CP phase.
- The eigen values of the neutrino mass matrix M_ν are obtained as $\lambda_1 = m_1$, $\lambda_2 = m_2 e^{2i\alpha}$, $\lambda_3 = m_3 e^{2i(\beta+\delta)}$.
- The elements of the neutrino mass matrix M_ν can be expressed as $M_{ab} = \sum_{i=1}^{3} U_{ai} U_{bi} \lambda_i$
- The condition for one vanishing minor can be given by

$$C_{mn} = (-1)^{m+n}(M_{\nu(ab)} M_{\nu(cd)} - M_{\nu(ef)} M_{\nu(gh)}) = 0 \tag{2}$$

  where C_{mn} is the cofactor of the elements of M_ν.

- There are six textures of neutrino mass matrices with one vanishing minor out of which we have studied here only one case $C_{11} = 0$, $m_{22}m_{33} - m_{23}m_{32} = 0$
- We now impose the two constrained equations of one vanishing minor and vanishing trace

$$\lambda_1\lambda_2 A_3 + \lambda_2\lambda_3 A_1 + \lambda_3\lambda_1 A_2 = 0, \lambda_1 + \lambda_2 + \lambda_3 = 0 \tag{3}$$

where $A_i = (U_{pj}U_{qj}U_{rk}U_{sk} - U_{tj}U_{uj}U_{vk}U_{wk}) + (j \longleftrightarrow k)$ here (i,j,k) is a cyclic permutation of (1,2,3).

- From equation (3) we have

$$X = \frac{m_2}{m_1}e^{2i\alpha} = \frac{(A_3 - A_1 - A_2) \pm \sqrt{(A_3 - A_1 - A_2)^2 - 4A_1A_2}}{2A_1} \tag{4}$$

$$Y = \frac{m_3}{m_1}e^{2i\beta} = \frac{(A_2 - A_1 - A_3) \mp \sqrt{(A_3 - A_1 - A_2)^2 - 4A_1A_2}}{2A_1}e^{-2i\delta} \tag{5}$$

- The ratios of the magnitude of the neutrino masses are

$$\rho = |\frac{m_2}{m_1}e^{2i\alpha}|, \sigma = |\frac{m_3}{m_1}e^{2i\beta}| \tag{6}$$

Now ρ and σ are related to each other with the ratio of solar and atmospheric mass squared difference R_ν given by

$$R_\nu = \frac{\delta m^2}{\Delta m^2} = \frac{2(\rho^2 - 1)}{2\sigma^2 - \rho^2 - 1} \tag{7}$$

where $\delta m^2 = m_2^2 - m_1^2$ and $\Delta m^2 = |m_3^2 - \frac{1}{2}(m_2^2 + m_1^2)|$

- It is observed that the pair (X_+, Y_-) and (X_-, Y_+) satisfies eq(3). Therefore we carry out our study using these two pairs.

Case $C_{11} = 0$

On plotting the graph for α and β for pair (X_+, Y_-) and (X_-, Y_+) we get

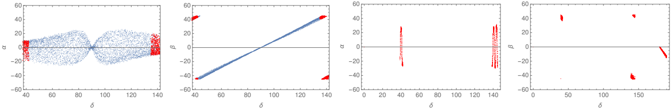

Figure 1: The first two figures are for the pair (X_+, Y_-) and the last two figures are for (X_-, Y_+). Blue: For NH, Red: For IH.

Symmetry realization

- We assumed simple form of M_R and M_D to produce generic M_ν which satisfies the constraints

$$M_R = \begin{pmatrix} 0 & x & -x \\ x & z & u \\ -x & u & v \end{pmatrix}, M_D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & ib \end{pmatrix}, M_\nu = M_D M_R^{-1} M_D^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta & X \\ 0 & X & -\Delta \end{pmatrix}$$

- General lagrangian [2] that producing to lepton masses are given by

$$\mathscr{L} = \left(\frac{<\Phi>}{\Lambda}\right)^{Q_{Di}+Q_{lj}} y_{ij}^{(k)} \overline{D}_i \phi_k l_{Rj} + \left(\frac{<\Phi>}{\Lambda}\right)^{Q_{Di}+Q_{\nu j}} y_{ij}^{(k)} \overline{D}_i \tilde{\phi}_{(k)} \nu_{Rj} + \left(\frac{<\Phi>}{\Lambda}\right)^{Q_{\nu Ri}+Q_{\nu R_j}} y_{ij}^{(k)} \chi_k \overline{\nu}_{Ri} \nu_{Rj} + h.c$$

The $Q_\alpha(\alpha = D, l_R, \nu_R)$ are the FN charges for the SM fermion ingredients under which different generations may be charged differently. The flavon Φ obtains the vaccum(VEV) $< \Phi >$ that breaks the FN symmetry. We assign FN charges for the lepton charges as

$$\overline{D}_{1,2,3} : (a+1, a, a),\ l_{R1,2,3} : (0, 1, 2),\ \nu_{R1,2,3} : (d, b, b)$$

- Now we impose $Z_8 \times Z_2$ symmetry under which the relevant particle fields transform as
  Symmetry under Z_8
  $\nu_{R1} \to \omega\nu_{R1}, \nu_{R2} \to \omega^2\nu_{R2}, \nu_{R3} \to \omega^4\nu_{R3}, \overline{D}_{L1} \to \omega\overline{D}_{L1}$, $\overline{D}_{L2} \to \omega^2\overline{D}_{L2}, \overline{D}_{L3} \to \overline{D}_{L3}, l_{R1} \to \omega^7 l_{R1}, l_{R2} \to \omega^2 l_{R2}, l_{R3} \to l_{R3}, \chi_1 \to \omega^5\chi_1, \chi_2 \to \omega^2\chi_2, \chi_3 \to \omega^4\chi_3, \chi_4 \to \chi_4, \phi_1 \to \omega^4\phi_1, \phi_2 \to \phi_2, \phi_3 \to \phi_3$
  Symmetry under Z_2
  $\nu_{R1} \to \nu_{R1}, \nu_{R2} \to -\nu_{R3}, \nu_{R3} \to \nu_{R2}$,
  $\overline{D}_{L1} \to \overline{D}_{L1}, \overline{D}_{L2} \to -\overline{D}_{L3}, \overline{D}_{L3} \to -i\overline{D}_{L2} l_{R1} \to l_{R1}, l_{R2} \to -l_{R3}, l_{R3} \to il_{R2}$
  $\chi_1 \to \chi_1, \chi_2 \to \chi_2, \chi_3 \to \chi_4, \chi_4 \to \chi_3, \phi_1 \to \phi_2, \phi_2 \to \phi_1$,
  $\phi_3 \to \phi_3$
  Forming the required bilinears dictated by Z_8 symmetry we obtain
  $\nu_{Ri}^T\nu_{Rj} = \begin{pmatrix} \omega^2 & \omega^3 & \omega^5 \\ \omega^3 & -1 & \omega^6 \\ \omega^5 & \omega^6 & 1 \end{pmatrix}, \overline{D}_{Li}\nu_{Rj} = \begin{pmatrix} \omega^2 & \omega^3 & \omega^5 \\ \omega^3 & -1 & \omega^6 \\ \omega & \omega^2 & -1 \end{pmatrix}, \overline{D}_{Li} l_{Rj} = \begin{pmatrix} 1 & \omega & \omega^7 \\ \omega^3 & -1 & \omega^2 \\ \omega & \omega^2 & 1 \end{pmatrix}$
  The lagrangian dictated by Z_8 is

$$\mathscr{L}_M^{Z_8} = Y_{\chi 12}^1 \chi_1 \nu_{R1}^T c^{-1}\nu_{R2} + Y_{\chi 23}^2 \chi_2 \nu_{R2}^T c^{-1}\nu_{R3} + Y_{\chi 13}^1 \tilde{\chi_1} \nu_{R1}^T c^{-1}\nu_{R3} + Y_{\chi 22}^3 \chi_3 \nu_{R2}^T c^{-1}\nu_{R2} + Y_{\chi 33}^4 \chi_4 \nu_{R3}^T c^{-1}\nu_{R3} + Y_{D_{22}}^1 \tilde{\phi_1}\overline{D}_{L2}\nu_{R2} + Y_{D_{33}}^2 \tilde{\phi_2}\overline{D}_{L3}\nu_{R3} + Y_{l_{11}}^3 \phi_1\overline{D}_{L1} l_{R1} + Y_{l_{22}}^1 \phi_1\overline{D}_{L2} l_{R2} + Y_{l_{33}}^2 \phi_2\overline{D}_{L3}\nu_{R3} \quad (8)$$

$$\mathscr{L}_M^{Z_8} \xrightarrow{Z_2} -Y_{\chi 12}^1 \tilde{\chi_1} \nu_{R1}^T c^{-1}\nu_{R3} + Y_{\chi 23}^2 \chi_2 \nu_{R3}^T c^{-1}\nu_2 - Y_{\chi 13}^1 \chi_1 \nu_{R1}^T c^{-1}\nu_{R2} + Y_{\chi 22}^3 \chi_4 \nu_{R3}^T c^{-1}\nu_{R3} + Y_{\chi 33}^4 \chi_3 \nu_{R2}^T c^{-1}\nu_{R2} - iY_{D_{22}}^1 \tilde{\phi_2}\overline{D}_{L3}\nu_{R3} + iY_{D_{33}}^2 \tilde{\phi_1}\overline{D}_{L2}\nu_{R2} + Y_{l_{22}}^1 \phi_2\overline{D}_{L3} l_{R3} + Y_{l_{11}}^3 \phi_3\overline{D}_{L1} l_{R1} + Y_{l_{33}}^2 \phi_1\overline{D}_{L2}\nu_{R2} \quad (9)$$

$Z_8 \times Z_2$ implies the following constraints on the Yukawa coupling.
$Y^1_{\chi 12} = -Y^1_{\chi 13}$, $Y^2_{\chi 23} = Y^2_{\chi 23}$, $Y^1_{\chi 13} = -Y^1_{\chi 12}$, $Y^3_{\chi 22} = Y^4_{\chi 33}$, $Y^4_{\chi 33} = Y^3_{\chi 22}$, $Y^1_{D12} = iY^2_{D33}$, $-iY^1_{D22} = Y^2_{\chi 33}$, $Y^3_{l11} = Y^3_{l11}$, $Y^1_{l_{22}} = Y^2_{l33}$, $Y^1_{l22} = Y^2_{\chi 33}$
Therefore M_ν turn out to be

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{v_{\chi 1}^2 v_{\phi 1}^2 y_{D_{22}}^2 y_{\chi 12}^2 \epsilon^{2a}}{v{\chi 1}^2 v{\chi 4} y_{\chi 12}^2 y_{\chi 22} + 2v{\chi 1}^2 v{\chi 2} y_{\chi 12}^2 y_{\chi 23} + v{\chi 1}^2 v{\chi 3} y_{\chi 12}^2 y_{\chi 33}} & \frac{i v_{\chi 1}^2 v_{\phi 1}^2 y_{D_{22}}^2 y_{\chi 12}^2 \epsilon^{2a}}{v{\chi 1}^2 v{\chi 4} y_{\chi 12}^2 y_{\chi 22} + 2v{\chi 1}^2 v{\chi 2} y_{\chi 12}^2 y_{\chi 23} + v{\chi 1}^2 v{\chi 3} y_{\chi 12}^2 y_{\chi 33}} \\ 0 & \frac{i v_{\chi 1}^2 v_{\phi 1}^2 y_{D_{22}}^2 y_{\chi 12}^2 \epsilon^{2a}}{v{\chi 1}^2 v{\chi 4} y_{\chi 12}^2 y_{\chi 22} + 2v{\chi 1}^2 v{\chi 2} y_{\chi 12}^2 y_{\chi 23} + v{\chi 1}^2 v{\chi 3} y_{\chi 12}^2 y_{\chi 33}} & \frac{-v_{\chi 1}^2 v_{\phi 1}^2 y_{D_{22}}^2 y_{\chi 12}^2 \epsilon^{2a}}{v{\chi 1}^2 v{\chi 4} y_{\chi 12}^2 y_{\chi 22} + 2v{\chi 1}^2 v{\chi 2} y_{\chi 12}^2 y_{\chi 23} + v{\chi 1}^2 v{\chi 3} y_{\chi 12}^2 y_{\chi 33}} \end{pmatrix}$$

Results and Conclusion

- For case $C_{11} = 0$ we observe that for (X_+, Y_-), for the allowed range of δ, $\alpha = (-25^o, 25^o)$ and $\beta = (-45^o, 45^o)$ for NH. While in case of IH, $\alpha = (-20^o, 20^o)$ and $\beta = (-45^o, -40^o) \oplus (40^o, 45^o)$. Since this case is allowed only for IH for the pair (X_-, Y_+),we observe that $\alpha = (-40^o, 40^o)$ and $\beta = (-35^o, -45^o) \oplus (-15^o, 0) \oplus (25^o, 40^o)$. We have done the symmetry realization of case $C_{11} = 0$ using FN mechanism and $Z_8 \times Z_2$ symmetry group.

References

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