12TH BIENNIAL PANE CONFERENCE

Masses of Heavy Flavour Mesons in a potential Model Approach

with Wave Function containing Airy's Infinite Series.

Abdul Aziz

Department of Physics

KARIMGANJ COLLEGE

Karimganj :: Assam

&

Department of Physics

Assam University, Silchar

ABSTRACT

We report the masses of B and D sectors heavy-flavoured mesons obtained by using our recently developed meson wave function employing potential model approach with linear confinement term in potential as parent in the perturbation method. As the wave-function involves infinite Airy's polynomial series, in carrying out the mass calculation, to avoid divergences, we have introduced some cut-off parameter for inter-quark separation. Our results for ground state masses of heavy-flavoured B and D sector mesons are reasonably closer to the PDG masses.

INTRODUCTION

- For the past couple of decades formulation of phenomenological model has proved to be very effective and reliable tool to understand, the salient features of QCD.
- In the infrared(Low energy) regime study of physical properties of hadrons containing heavy quark is challenging and potential model approach has proved to be very successful.
- Such model may not be derived directly from fast principle but include its basic features.

Background and Motivation

- Mesons are quark-antiquark bound system, heavy flavoured mesons consist of one heavy quark/antiquark (*c*, *b*, *t*) and a light antiquark/quark *u*,*d*,*s*). Weak decays of heavy-flavoured mesons [1] gives information on the standard model of electroweak interaction and internal structure of hadrons like weak mixing angles, CKM matrix elements, exploring CP violation.
- Study of such mesons gives information on QCD at low energy, which is not directly explorable. Formulation of phenomenological potential models [2] has proved to be very effective and reliable tool in this regard.

1. N. Isgur and M. B. Wise. Phys. Rev. Lett., 66, 1130(1991).
2. E. Eichten. Third international conference at Vanderbilt university. New Results in High Energy Physics, Nashville, Tennessee, March 6-8(1978).

MASSES OF MESONS

This work reports the studies on the masses of Heavy-Flavoured Mesons in Potential Model approach with Wave Function containing Airy's Infinite Series. Masses of B and D sectors heavy-flavoured mesons are obtained by using recently developed meson wave function obtained in potential model approach with linear confinement term in potential as parent in the perturbation method. Airy's infinite series in potential comes if we take linear term in Cornell potential as parent and Coulombic term as perturbation.

Pseudoscalar meson mass can be computed from the following relation

$$M_P = m_Q + m_{\overline{Q}} + \Delta E$$

$$where,\ \Delta E = < H >$$

Hamiltonian operator H has the form:

$$H = -\frac{\nabla^2}{2\mu} + V(r)$$

The Cornell potential is:

$$V(r) = -\frac{4\alpha_s}{3r} + br + c$$

MASSES OF MESONS

$$\nabla^2 \equiv \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2}$$

For ground state meson ($l = 0$),

$$\nabla^2 \equiv \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$$

The Cornell potential is:

$$V(r) = -\frac{4\alpha_s}{3r} + br + c$$

Meson wave function is obtained from two-body Schrodinger equation by applying quantum mechanical perturbation technique. With linear confinement term (*br*) in potential as parent in perturbation method, the wave function comes out in terms of Airy's infinite series.

MASSES OF MESONS

$$\Psi^{total}(r) = \frac{N'}{r}[1 + A_1(r)r + A_2(r)r^2 + A_3(r)r^3 + ...]A_i[\varrho_1 r + \varrho_0]$$

Here, we have taken $\varrho_1 = (2\mu b)^{1/3}$ and $\varrho(r) = \varrho_1 r + \varrho_0$. N' is the normalization constant of total wave function which is having the dimension of $GeV^{1/2}$.
Considering relativistic effect on the wave function following Dirac modification, the total relativistic wave function is expressed as:

$$\Psi^{tot}_{rel}(r) = \frac{N'}{r}[1 + A_1(r)r + A_2(r)r^2 + A_3(r)r^3 + ...]A_i[\varrho_1 r + \varrho_0](\frac{r}{a_0})^{-\epsilon}$$

with,

$$a_0 = \frac{3}{4\mu\alpha_s} = \frac{1}{B\mu} \quad and \quad \epsilon = 1 - \sqrt{1 - (\frac{4\alpha_s}{3})^2} = 1 - \sqrt{1 - (B)^2}$$

It is to be mentioned that, the Airy's infinite series as a function of $\varrho = \varrho_1 r + \varrho_0$ can be expressed as

$$A_i[\varrho] = a_1[1 + \frac{\varrho^3}{6} + \frac{\varrho^6}{180} + \frac{\varrho^9}{12960} + ...] - b_1[\varrho + \frac{\varrho^4}{12} + \frac{\varrho^7}{504} + \frac{\varrho^{10}}{45360} + ...]$$

with $a_1 = \frac{1}{3^{2/3}\Gamma(2/3)} = 0.3550281$ and $b_1 = \frac{1}{3^{1/3}\Gamma(1/3)} = 0.2588194$.

MASSES OF MESONS

In the wave function mentioned above, considering only up to the term $A_2(r)$ in the first infinite series, we have calculated meson masses.

$$< H >=< -\frac{\nabla^2}{2\mu} > + < -\frac{4\alpha_s}{3r} > + < \sigma r > + < c >$$
$$=< H_1 > + < H_2 > + < H_3 > + < H_4 >$$

$$< H_1 >= \int_0^\infty 4\pi r^2 \Psi(r)[-\frac{\nabla^2}{2\mu}]\Psi(r)dr$$
$$= \int_0^\infty 4\pi r^2 \Psi(r)[-\frac{1}{2\mu}(\frac{d^2\Psi(r)}{dr^2} + \frac{2}{r}\frac{d\Psi(r)}{dr})]dr$$
$$< H_2 >= \int_0^\infty 4\pi r^2 (-\frac{B}{r})|\Psi(r)|^2 dr = -4\pi B \int_0^\infty r|\Psi(r)|^2 dr$$
$$< H_3 >= \int_0^\infty 4\pi r^2 (br)|\Psi(r)|^2 dr = 4\pi b \int_0^\infty r^3|\Psi(r)|^2 dr$$
$$< H_4 >= \int_0^\infty 4\pi r^2 (c)|\Psi(r)|^2 dr = 4\pi c \int_0^\infty r^2|\Psi(r)|^2 dr$$

MASSES OF MESONS

With the expressions for $< H >$ I have proceeded to calculate pseudoscalar meson masses. Here, the strong coupling constant αs is take as 0.39 and 0.22 at charm and bottom mass scale, the value of the confinement parameter b is 0.183 GeV^2 and the scale factor c in the potential is taken as $1GeV$ as in our previous cases, to make it compatible with meson masses in the calculations. The input values for quark masses are taken from ***A. K. Rai, B. Patel and P. C. Vinodkumar, Phy. Rev C 78 055202(2008)***. The masses for pseudoscalar B and D mesonsare taken from PDG.

Now, it is worth mentioning here that the wave function containing infinite Airy's polynomial series brings in divergences while carrying out the integrations with infinite upper limit as in the calculation of different terms involved in $< H >$. This has put forward the compulsion to consider some reasonable cut-off to the upper infinite integration limit. In principle, this cut-off limit should be within the range of size of hadrons ($1/Mp$, Mp being the hadron mass). In our calculations we fix this cut-off out from the convergence condition of total wave function obtained through perturbation technique.

MASSES OF MESONS

$$\Psi'(r) < \Psi(r) \;\; i.e, \; |A_1(r)r + A_2(r)r^2 + \cdots\cdots| < 1$$

This condition gives us the limiting values of cut-off parameter r_0 for different mesons. Considering upto the term $A_2(r)$ in the series, we obtain the cut-off limit ultimately from:

$$B_1r^3 + B_2r_0^4 + B_3r_0^5 + B_4r_0^6 = 0$$

The numerical values of this cut-off r_0 for different mesons are shown in Table-3. As, perturbed eigen energy E and unperturbed energy W' are involved in the calculations of r_0, these are also reported in the same table. The Table-3 shows that the cut-off value should be greater than 3 GeV^{-1}. Consideration of this cut-off will not sacrifice the expected value or nature of the integrands; this is because Airy's function Ai falls very sharply with r beyond $r = 3$ ($Ai[3] = 0.0066$).

With that cut-off, then we calculate the energy expectation value ΔE for different mesons and compute the corresponding masses numerically. Results are shown in Table-4, in which we also record the ratio $\frac{\Delta E}{M_p}$ measured from our calculations. In Figure-1, we plot our masses along with standard PDG masses.

MASSES OF MESONS

Table 1: Quark masses from ref

Quark	Mass)
$m_{u/d}$	0.18 GeV
m_s	0.25 GeV
m_c	1.31 GeV
m_b	4.66 GeV

A. K. Rai, B. Patel and P. C. Vinodkumar, Phy. Rev C 78 055202(2008).

Table 2: Meson masses from PDG(2012)

Meson	Mass(PDG) in GeV
$D^0(c\overline{u})$	1.8649
$D_s(c\overline{s})$	1.9685
$B^0(d\overline{b})$	5.2796
$B_s(s\overline{b})$	5.3668

J. Beringer *et al* (Particle Data Group), Phy. Rev. D86 010001(2012)(http://pdg.ibi.gov).

MASSES OF MESONS

Table 3: Cut-off for different mesons

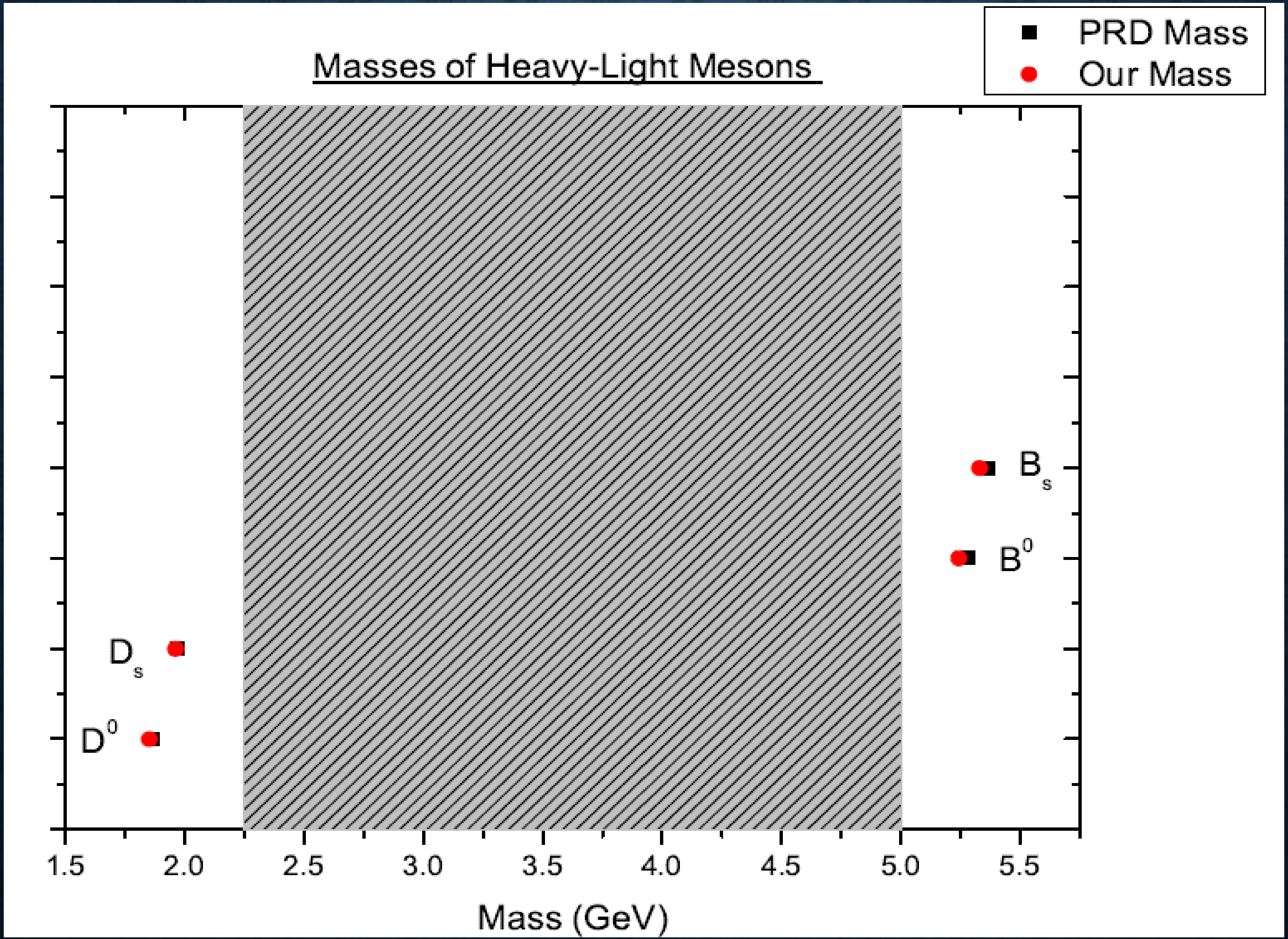
Meson	E (GeV)	W'(GeV)	$r_0(GeV^{-1})$
D^0	0.8428	0.4703	3.6052
D_s	0.7659	0.3867	3.5362
B^0	0.8066	0.4264	3.5783
B_s	0.7215	0.3367	3.5021

Table 4: Our calculated Meson masses (in GeV)

Meson	$< H >= \triangle E$	Our mass(M_p)	PDG mass	$\triangle E/M_p$
$D^0(c\overline{u})$	0.3603	1.8503	1.8649	0.1947
$D_s(c\overline{s})$	0.3997	1.9597	1.9685	0.2040
$B^0(d\overline{b})$	0.4010	5.2410	5.2796	0.0765
$B_s(s\overline{b})$	0.4194	5.3294	5.3668	0.0787

MASSES OF MESONS

Our calculated meson masses



CONCLUSION

In this work, we have developed formalism for the studies of meson masses. As the wave function contains infinite Airy series along with another infinite polynomial series, in the calculation of mass we consider some truncated series. Also, to avoid divergence arising due to infinite Airy series in calculations, we here introduce reasonable cut-off to infinite upper limit of integrations involved.

Our results for ground state masses of heavy-light mesons are closer to the expectation (PDG masses). However, there is scope of refinement by considering higher terms in the wave function in the calculation. Also, the formalism may also be extended to measure masses of higher spectroscopic states of mesons.

This work is under communication.

THANK YOU ALL